CHELTENHAM GIRLS'
HIGH SCHOOL

2021 Assessment Task 4

Mathematics Extension 1

General Instructions

- No reading time
- Working time – 1 hour and 30 minutes (90 minutes)
- Time allocated for uploading of attachments – 15 minutes
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- In Questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 51

Section I – 5 marks

- Attempt Questions 1 – 5
- Allow about 10 minutes for this section

Section II – 46 marks

- Attempt Questions 6 – 9
- Questions are NOT of equal value
- Allow about 1 hour and 20 minutes for this section

Section I

5 marks
Attempt Questions 1–5
Allow about 10 minutes for this section

Q1

A particle is initially positioned at $x = -2$. Its motion has velocity of $v = \dfrac{1}{t+e}$.

Which of the following will be true when the particle reaches the origin?

A. $t = 2, v = \dfrac{1}{2+e}$

B. $t = e, \dot{x} = \dfrac{1}{2e}$

C. $t = e^2, v = \dfrac{1}{e^2+e}$

D. $t = e^3 - e, v = e^{-3}$

Q2

A company has a board of twelve directors. Six of these were selected at random to be candidates in the election for the positions of President, Vice-President and Treasurer.

In how many ways can these three senior positions be filled?

A. ${}^{12}\mathbf{P}_3$

B. $\dfrac{12!}{3!}$

C. $\dfrac{{}^{12}\mathbf{P}_6}{3!}$

D. $\dfrac{{}^{12}\mathbf{P}_6}{3!3!}$

Q3

Peter draws a vector from the origin to the point $A(-2, 5)$. He then draws a vector $-3\underset{\sim}{\mathbf{i}} + 2\underset{\sim}{\mathbf{j}}$ from A, ending at point B.

How far is point B from the origin?

A. $\sqrt{10}$ units

B. $\sqrt{74}$ units

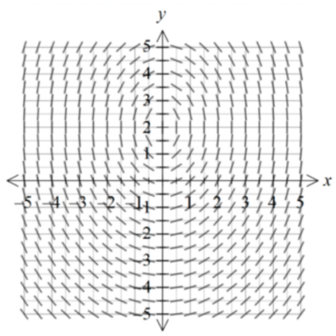
C. $\sqrt{13}$ units

D. $\sqrt{34}$ units

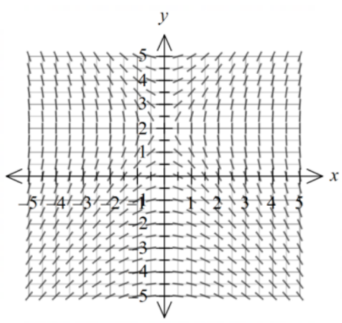
Q4

Which of the following best represents the direction field for the differential equation $\frac{dy}{dx} = \frac{-2x}{y-2}$?

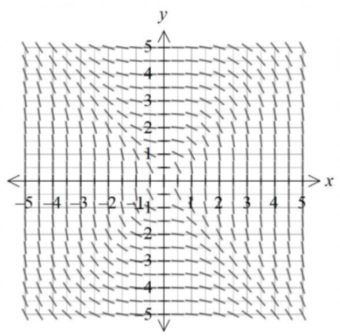
A.



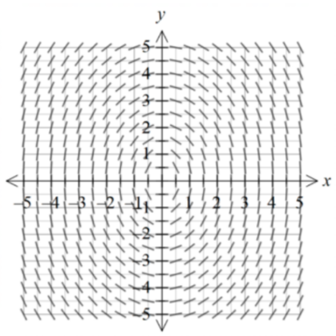
B.



C.



D.



Q5

If $\boldsymbol{a} = \binom{k}{3}$ and $\boldsymbol{b} = \binom{6}{k-6}$. Find the value of k when vector **a** is perpendicular to vector **b**.

A. 1

B. -1

C. 2

D. -2

Section II
48 marks
Attempt Questions 6-9
Allow about 1 hour and 20 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (11 marks)

(a) Let $P(x) = x^3 + 5x^2 + x - 15$.

(i) Show that $P(-3) = 0$. **1**

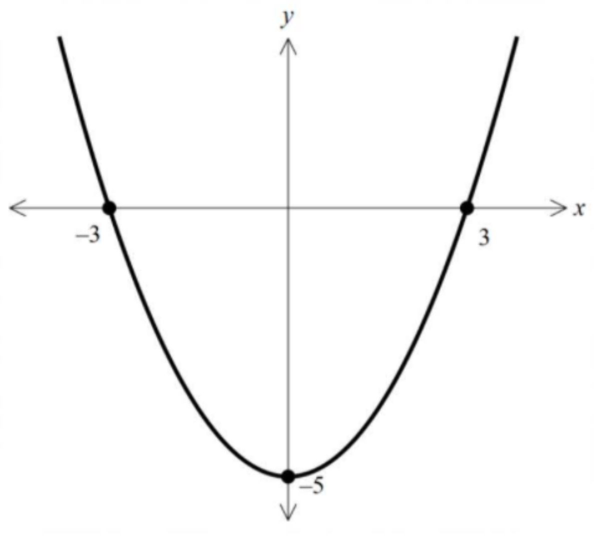
(ii) Hence, factorise the polynomial $P(x)$ as $A(x)B(x)$, where $B(x)$ is a quadratic polynomial. **2**

(b) For what values of x is $\dfrac{x}{x-1} \geq -2$? **3**

(c) Find the angle between the vectors $\underset{\sim}{\mathbf{a}} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$ and $\underset{\sim}{\mathbf{b}} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$, correct to the nearest degree. **2**

Question 6 continues

(d) The diagram shows the graph of $y = f(x)$.



(i) Sketch $y = f^{-1}(x)$ after making an appropriate restriction on the domain of $f(x)$. **1**

(ii) Sketch $y = \dfrac{1}{f^{-1}(x)}$. **2**

End of Question 6

Question 7 (11 marks)

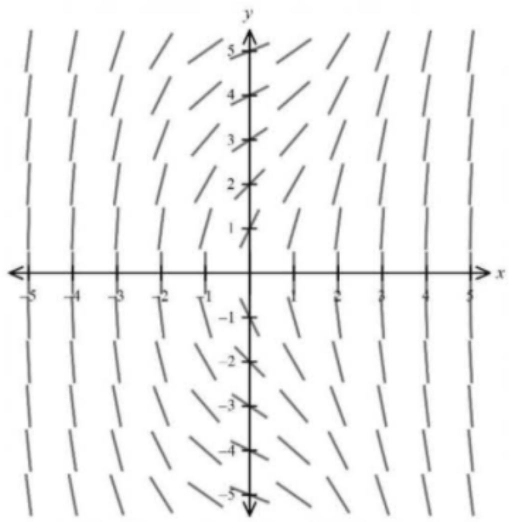
(a) Use the principle of mathematical induction to show that for all integers $n \geq 1$, **3**

$$1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2}{4}(n+1)^2 \ .$$

(b) Consider the differential equation: $\dfrac{dy}{dx} = \dfrac{3x^2+4}{2y}$.

(i) Find a solution to the equation in the form $y = f(x)$. **2**

(ii) A direction field diagram for the differential equation is shown below. **2**
Use it to sketch two possible solution curves, one through $(-1, 0)$ and the other through $(2, 0)$.



(c) Consider the binomial expansion $\binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \ldots + \binom{n}{r}x^r + \ldots + \binom{n}{n}x^n = (1+x)^n$.

(i) Show that $1 - \binom{n}{1} + \binom{n}{2} - \ldots + (-1)^n\binom{n}{n} = 0$ **1**

(ii) Show that $1 - \frac{1}{2}\binom{n}{1} + \frac{1}{3}\binom{n}{2} - \ldots + (-1)^n\frac{1}{n+1}\binom{n}{n} = \frac{1}{n+1}$ **3**

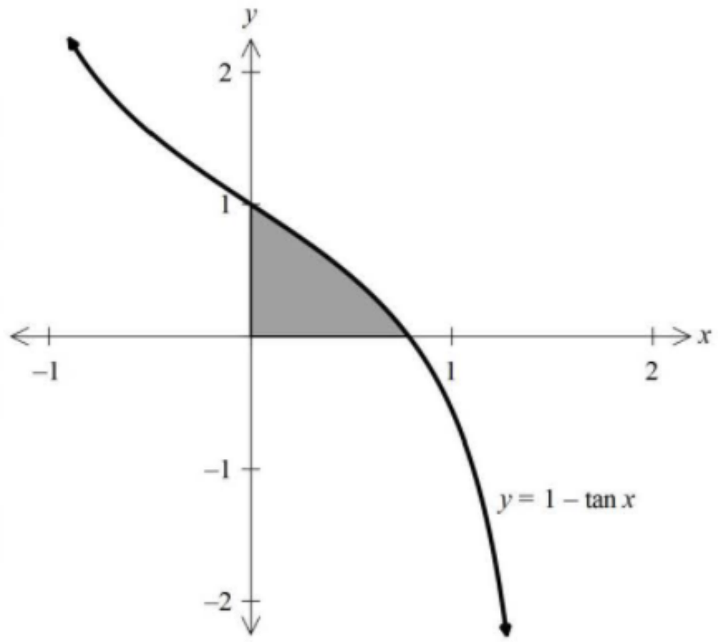
End of Question 7

Question 8 (15 marks)

(a)

(i) Show that $\int \tan^2 x \ dx = \tan x - x + C$. **1**

(ii) The region bounded by the curve $y = 1 - \tan x$, the y-axis and the x-axis is shaded below. If this region is rotated about the x-axis, find the exact volume of the solid formed. **3**



(b) Solve the equation $\cos 3x - \cos 2x + \cos x = 0$ for $0 \le x \le 2\pi$. **3**

(c) (i) Differentiate $x \cos^{-1} x$ with respect to x. **1**

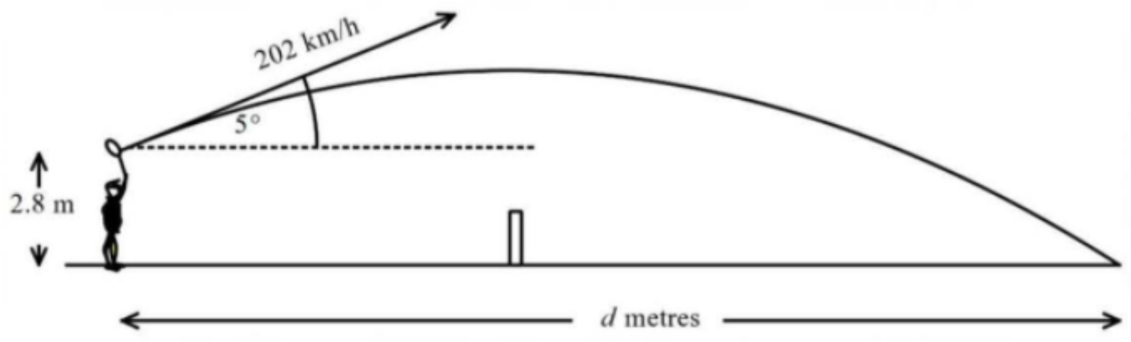
(ii) Hence find $\int \cos^{-1} x \ dx$ **2**

Question 8 continues

Question 8 (continued)

(d) Mick hits a tennis ball from a height 2.8 metres above the ground. The angle of elevation as the racket hits the ball is $5°$ and the velocity is 202 km/h.

The ball lands at a horizontal distance of d metres from Mick's feet.



(i) Using $g = 9.8 \text{ ms}^{-2}$ as acceleration due to gravity, show that the vector displacement of the tennis ball is: **3**

$$\underset{\sim}{\mathbf{s}}(t) \approx 55.8t\underset{\sim}{\mathbf{i}} + (4.9t - 4.9t^2 + 2.8)\underset{\sim}{\mathbf{j}}\ .$$

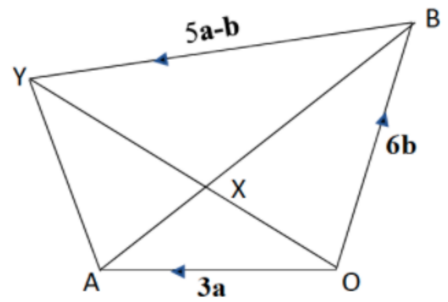
(ii) If d is greater than 26.2, the ball will be called "out". **2**

Find the value of d and determine if the ball will be called "out".

End of Question 8

Question 9 (9 marks)

(a)



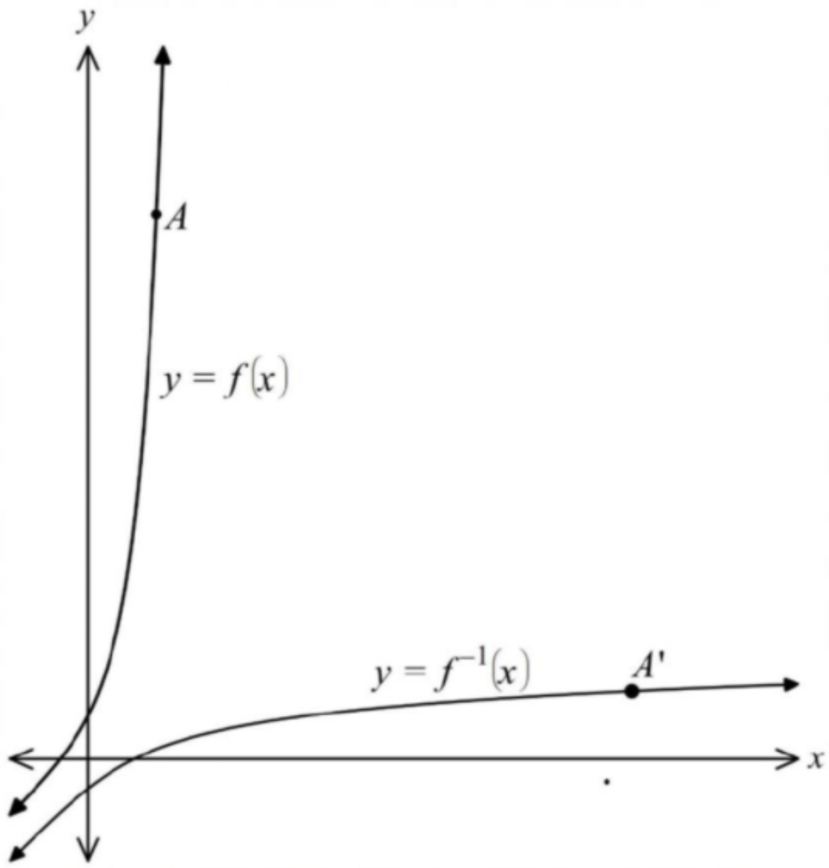
(i) Express $\overrightarrow{AB}$ in terms of **a** and **b.** **1**

(ii) X is the point on AB such that $AX : XB = 1 : 2$ and $\overrightarrow{BY} = 5\boldsymbol{a} - \boldsymbol{b}$. **3**
Prove $\overrightarrow{OX} = \frac{2}{5}\overrightarrow{OY}$.

Question 9 continues

Question 9 (continued)

(b) Consider the graphs of $y = f(x)$ and $y = f^{-1}(x)$ where $f(x) = 1 + x + e^x$ shown below.



The point A lies on $y = f(x)$ and has an x-coordinate of 3. The point A' is the image of point A on $y = f^{-1}(x)$.

(i) Find the exact coordinates of A'. **1**

(ii) Show that the equation of the tangent to $y = f^{-1}(x)$ at A' is $y = \dfrac{x - 1 + 2e^3}{1 + e^3}$. **2**

(iii) Find the coordinates of the point where the tangent to $y = f(x)$ at A meets the tangent from part (ii). **2**

End of paper

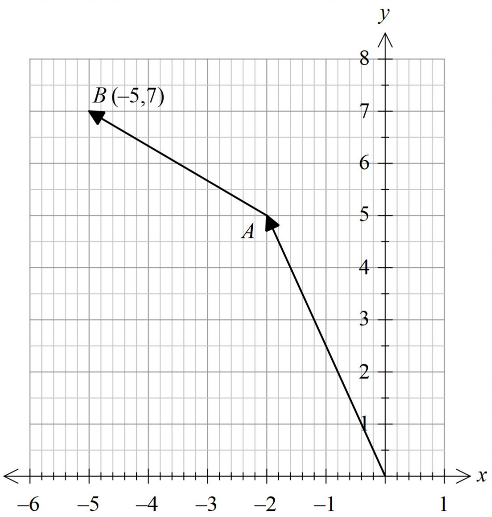
Cheltenham Girls' High School

2021 | TRIAL HSC EXAMINATION

Mathematics Extension 1 Solutions

Section I

No	Working	Answer
1.	$\dot{x} = \dfrac{1}{t+e}$ $x = \ln(t+e) + C$ when $t = 0, x = -2$ $-2 = \ln(e) + C$ $-2 = 1 + C$ $C = -3$ $\therefore \quad x = \ln(t+e) - 3$ when $x = 0$: $0 = \ln(t+e) - 3$ $\ln(t+e) = 3$ $e^3 = t + e$ $t = e^3 - e$ when $t = e^3 - e$: $v = \dfrac{1}{e^3 - e + e}$ $= e^{-3}$	D

No	Working	Answer
3.	 $-2\underset{\sim}{\mathbf{i}} + 5\underset{\sim}{\mathbf{j}} + \left(-3\underset{\sim}{\mathbf{i}} + 2\underset{\sim}{\mathbf{j}}\right) = -5\underset{\sim}{\mathbf{i}} + 7\underset{\sim}{\mathbf{j}}$ $\text{OB} = \sqrt{5^2 + 7^2}$ $= \sqrt{74}$ units	B
4.	There are ${}^{12}\mathbf{C}_6$ ways of selecting the six directors to go through to the elections. There are ${}^{6}\mathbf{P}_3$ ways of electing people to the three positions. ${}^{12}\mathbf{C}_6 \times {}^{6}\mathbf{P}_3 = \frac{12!}{6!6!} \times \frac{6!}{3!}$ $= \frac{12!}{6!3!}$ $= \frac{{}^{12}\mathbf{P}_6}{3!}$	C

5.	$\boldsymbol{a}.\boldsymbol{b} = 0$ $(k \times 6) + \left(3 \times (k - 6)\right) = 0$ $6k + 3k - 18 = 0$ $9k = 18$ $\therefore k = 2$	C

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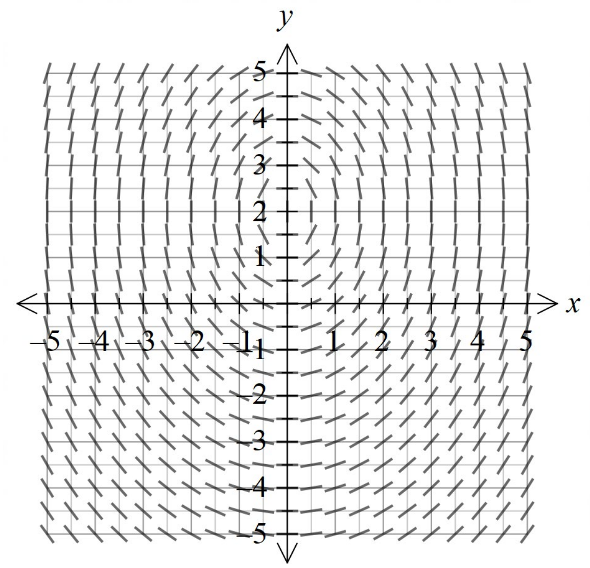
<td>2.</td>
<td>


By substituting coordinate values into $\frac{dy}{dx}$ we see that option A is the correct one.

$\text{e.g. Test } (3,0)$
$\frac{dy}{dx} = \frac{-2(3)}{0-2} = 3$
Only A has a positive gradient at $(3,0)$
est $(1,1)$
$\frac{dy}{dx} = \frac{-2(1)}{1-2} = 2$
Only A has a positive gradient at $(1,1)$
Also, as $y = 2$ makes denominator zero, would expect vertical gradient for points with $y = 2$ and as $x = 0$ makes numerator zero, would expect horizontal gradient for points with $x = 0$, which only happens with A and B, then eliminate B by testing a point.
</td>
<td>A</td>
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1.	A ○	B ○	C ○	D ●
2.	A ●	B ○	C ○	D ○
3.	A ○	B ●	C ○	D ○
4.	A ○	B ○	C ●	D ○
5.	A ○	B ○	C ●	D ○

[Comments]

Mathematics Extension 1 Trial HSC

Solutions 2021

Section II

Question	Working and answer	Marks	Mark Allocation
6.	(a) $P(x) = x^3 + 5x^2 + x - 15$ (i) $P(-3) = (-3)^3 + 5(-3)^2 + (-3) - 15$ $= -27 + 45 - 3 - 15$ $= 0$	1	1 mark for correct substitution
	(ii) Since $P(-3) = 0$, $(x + 3)$ is a factor of $P(x)$ $$\begin{array}{r} x^2 + 2x - 5 \\ x + 3 \overline{)\, x^3 + 5x^2 + x - 15} \\ \underline{x^3 + 3x^2} \\ 2x^2 + x \\ \underline{2x^2 + 6x} \\ -5x - 15 \\ -\underline{5x - 15} \\ 0 \end{array}$$ $\therefore P(x) = (x + 3)(x^2 + 2x - 5)$	2	2 marks for correct answer with appropriate method 1 mark for working towards solution

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(b) $\dfrac{x}{x-1} \geq -2$

$x \neq 1$

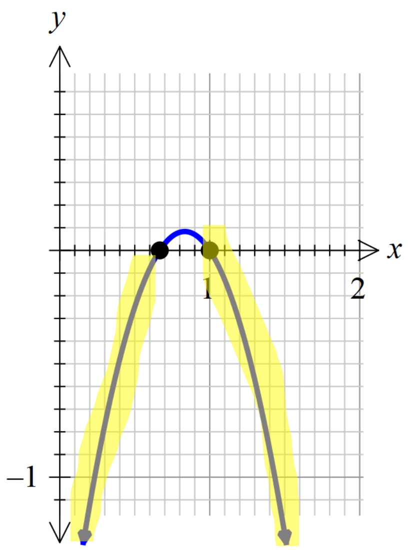
$x(x-1) \geq -2(x-1)^2$

$0 \geq -2(x-1)^2 - x(x-1)$

$0 \geq (x-1)[-2(x-1) - x]$

$0 \geq (x-1)(-3x+2)$

$x \leq \dfrac{2}{3}$ or $x > 1$



</td><td>3</td><td>

3 marks for correct solution with adequate reasoning

2 marks for correct critical values

1 mark for working towards solution using an appropriate method

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(c) $\underset{\sim}{\mathbf{a}} = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$ and $\underset{\sim}{\mathbf{b}} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$

$$\cos\theta = \frac{\underset{\sim}{\mathbf{a}}.\underset{\sim}{\mathbf{b}}}{|\underset{\sim}{\mathbf{a}}||\underset{\sim}{\mathbf{b}}|}$$

$$= \frac{(3 \times -1) + (-5 \times 4)}{\sqrt{3^2 + (-5)^2} \times \sqrt{(-1)^2 + 4^2}}$$

$$= \frac{-23}{\sqrt{578}}$$

$$\therefore \quad \theta = \cos^{-1}\left(\frac{-23}{\sqrt{578}}\right)$$

$$= 163 \cdot 0724869$$

$$\approx 163°$$

</td><td>2</td><td>

2 marks for correct answer

1 mark for some valid working towards correct answer

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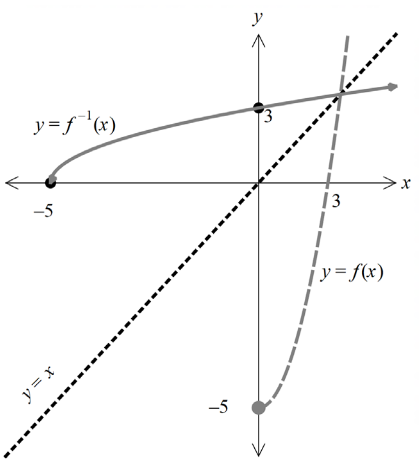
<td>(d)
(i) This sketch has the domain of f(x) restricted to $x \geq 0$.

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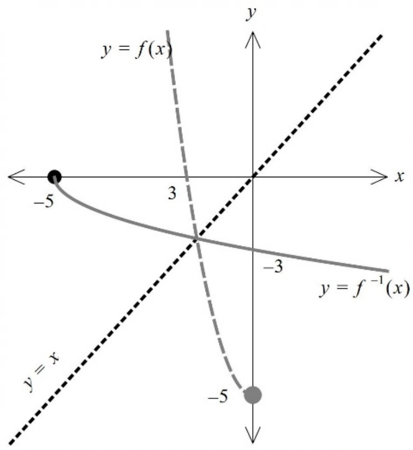
<td>Or if the domain of f(x) restricted to $x \leq 0$.

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<td>1</td>
<td>1 mark for sketch which identifies correct x and y intercepts for the inverse function</td>
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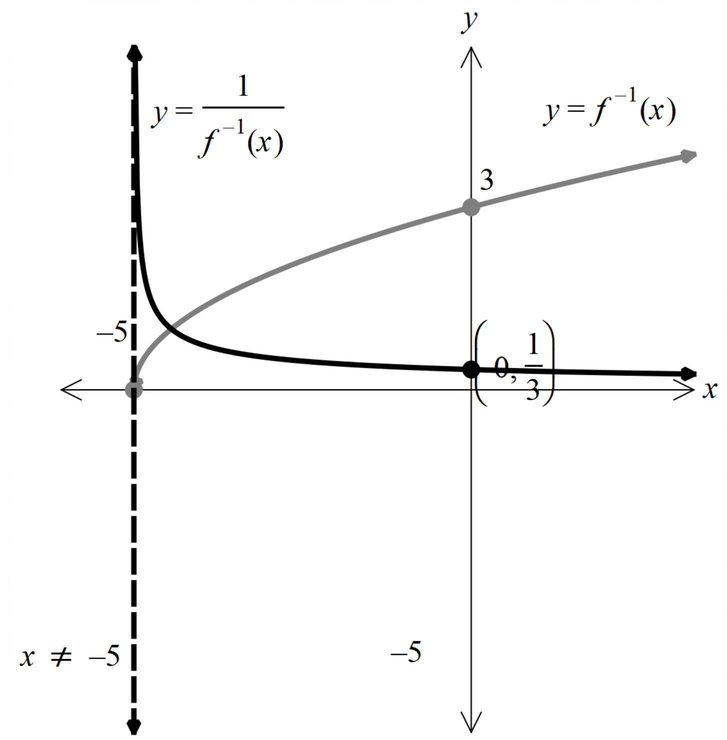
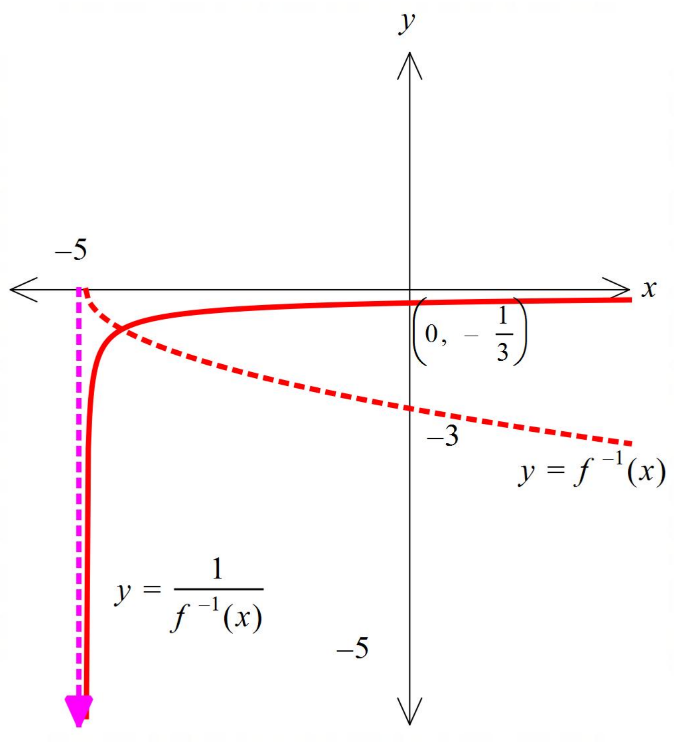
<td>(d) ii)


Or if using $x \leq 0$.

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<td>2</td>
<td>2 marks for correct graph showing correct y-intercept, asymptote at x=-5 and correct limits

1 mark for correct asymptote or y-intercept</td>
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<td>7.</td>
<td>(a) $1^3 + 2^3 + 3^3 + ... + n^3 = \frac{n^2}{4}(n+1)^2$

Step 1: Prove true for $n = 1$

$\text{LHS} = 1^3$
$= 1$
$\text{RHS} = \frac{1^2}{4}(1+1)^2$
$= \frac{1}{4} \times 4$
$= 1$
$= \text{LHS}$
$\therefore$ true for $n = 1$

Step 2: Assume true for $n = k$
$1^3 + 2^3 + 3^3 + ... + k^3 = \frac{k^2}{4}(k+1)^2$

Step 3: Prove true for $n = k + 1$

RTP: $1^3 + 2^3 + 3^3 + ... + k^3 + (k+1)^3 = \frac{(k+1)^2}{4}(k+2)^2$

$\text{LHS} = \frac{k^2}{4}(k+1)^2 + (k+1)^3$ (using statement from Step 2)
$= (k+1)^2\left[\frac{k^2}{4} + (k+1)\right]$
$= \frac{(k+1)^2}{4}(k^2 + 4k + 4)$
$= \frac{(k+1)^2}{4}(k+2)^2$
$= \text{RHS}$
$\therefore$ true for $n = k + 1$ when true for $n = k$.
Since proven true for $n = 1$, must be true for all $n \geq 1$.</td>
<td>3</td>
<td>3 marks for full proof, clearly showing true for $n = 1$ and for $n = k + 1$ when assumed true for $n = k$.

2 marks for proving true for $n = 1$ and for progress towards proof for $n = k + 1$.

1 mark for proving for $n = 1$ and no further merit</td>
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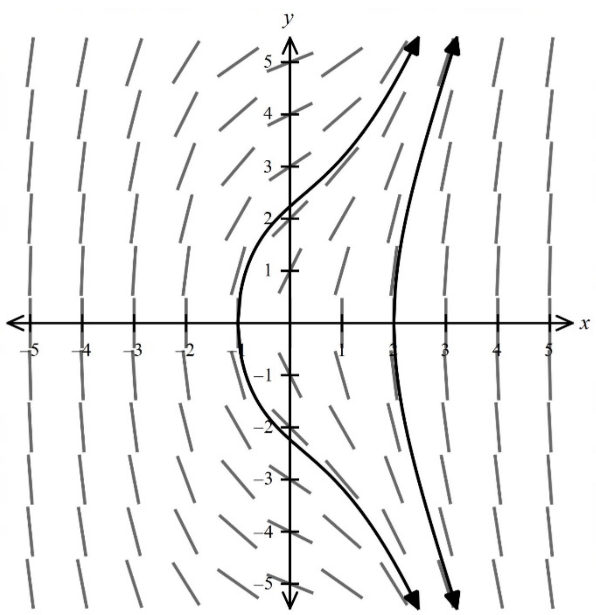
7 continued	(b) (i) $\frac{dy}{dx} = \frac{3x^2+4}{2y}$ $2y\frac{dy}{dx} = 3x^2+4$ $\int 2y\frac{dy}{dx}\,dx = \int 3x^2+4\,dx$ $\int 2y\,dy = \int 3x^2+4\,dx$ $y^2 = x^3+4x+C$ Possible solution is of the form $y = \pm\sqrt{x^3+4x+C}$	2	2 marks for correct answer (As it only asks for **a solution**, if C or the$\pm$ is left out, it is okay.) 1 mark for working towards an answer rearranging correctly
	(b) (ii) $\frac{dy}{dx} = \frac{3x^2+4}{2y}$ Note when $y = 0$, $\frac{dy}{dx}$ is undefined, so a vertical gradient on x-axis at x =-1 and at x = 2. . Follow slope lines to get shape of curve from there. If equation in (i) doesn't have $\pm$ then only section above x axis is the graph. (Don't penalise if student shows the whole grasph) 	2	2 marks for a diagram with 2 correct graphs 1 mark for one correct graph or two incorrect graphs with the same minor error

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continued</td><td>(c) (i)
$$\binom{n}{0}+\binom{n}{1}x+\binom{n}{2}x^2+\ \dots+\binom{n}{r}x^r+\ \dots+\binom{n}{n}x^n=(1+x)^n$$
Let $x=-1$
$$\binom{n}{0}+\binom{n}{1}-1+\binom{n}{2}(-1)^2+\ \dots+\binom{n}{n}(-1)^n=(1-1)^n$$
$1-\binom{n}{1}+\binom{n}{2}-\ \dots+(-1)^n\binom{n}{n}=0$ as required</td><td>1</td><td>1 mark for the correct proof</td></tr>
<tr><td></td><td>(ii) Integrate both sides with respect to x
$$x+\binom{n}{1}\frac{x^2}{2}+\binom{n}{2}\frac{x^3}{3}+\ \dots+\binom{n}{n}\frac{x^{n+1}}{n+1}=\frac{(1+x)^{n+1}}{n+1}+C$$
To find $C\ sub\ x=0$
$$0+\binom{n}{1}\frac{0^2}{2}+\binom{n}{2}\frac{0^3}{3}+\ \dots+\binom{n}{n}\frac{0^{n+1}}{n+1}=\frac{(1+0)^{n+1}}{n+1}+C$$
$0=\frac{(1)^{n+1}}{n+1}+C$
$0=\frac{1}{n+1}+C$
$\therefore C=\frac{-1}{n+1}$
$$\therefore x+\binom{n}{1}\frac{x^2}{2}+\binom{n}{2}\frac{x^3}{3}+\ \dots+\binom{n}{n}\frac{x^{n+1}}{n+1}=\frac{(1+x)^{n+1}}{n+1}-\frac{1}{n+1}$$
$when\ x=-1$
$$-1+\binom{n}{1}\frac{(-1)^2}{2}+\binom{n}{2}\frac{(-1)^3}{3}+\ \dots+\binom{n}{n}\frac{(-1)^{n+1}}{n+1}=\frac{(1-1)^{n+1}}{n+1}-\frac{1}{n+1}$$
$$-1+\binom{n}{1}\frac{1}{2}-\binom{n}{2}\frac{1}{3}+\ \dots+\binom{n}{n}\frac{(-1)^n(-1)^1}{n+1}=0-\frac{1}{n+1}$$
$swapping\ sides$
$$1-\binom{n}{1}\frac{1}{2}+\binom{n}{2}\frac{1}{3}+\ \dots-\binom{n}{n}\frac{(-1)^n}{n+1}=\frac{1}{n+1}$$
$$1-\frac{1}{2}\binom{n}{1}+\frac{1}{3}\binom{n}{2}-\ \dots+(-1)^n\frac{1}{n+1}\binom{n}{n}\ =\frac{1}{n+1}$$
as required</td><td>3</td><td>3 marks for the correct proof

2 marks for some progress towards the solution

1 mark for integrating both sides of the expression correctly or correct use of substitution at some point in the solution</td></tr>
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8.	(a) (i) $\sec^2 x = 1 + \tan^2 x$ $\tan^2 x = \sec^2 x - 1$ $\int \tan^2 x \ dx = \int \sec^2 x - 1 \ dx$ $= \tan x - x + C$	1	for valid proof
	(ii) x – intercept occurs at $1 - \tan x = 0$ $\tan x = 1$ $x = \frac{\pi}{4}$ $V = \pi \int y^2 \ dx$ $= \pi \int_0^{\frac{\pi}{4}} (1 - \tan x)^2 \ dx$ $= \pi \int_0^{\frac{\pi}{4}} 1 - 2 \tan x + \tan^2 x \ dx$ $= \pi \int_0^{\frac{\pi}{4}} 1 - 2 \frac{\sin x}{\cos x} + \tan^2 x \ dx$ $= \pi[x + 2 \ln(\cos x) + \tan x - x]_0^{\frac{\pi}{4}}$ $= \pi\left[\left(2 \ln\left(\cos \frac{\pi}{4}\right) + \tan \frac{\pi}{4}\right) - 2 \ln(\cos 0) + \tan 0\right]$ $= \pi\left(\left(2 \ln\left(\frac{1}{\sqrt{2}}\right) + 1\right) - 2 \ln 1\right)$ $= \pi\left(2\ln\left(\frac{1}{\sqrt{2}}\right) + 1\right)$ cubic units $= \pi\left(2\ln\left(2^{-\frac{1}{2}}\right) + 1\right)$ cubic units $= \pi(1 - \ln(2))$ cubic units	3	3 marks for correct answer (in form of any of last 3 lines) including integration of tan x and showing substitution 2 marks for correct integration and substitution with calculation error Or 2 marks for correct integration and substitution using incorrect x-intercept value or equivalent merit 1 mark for correct x-intercept or correct integration of tan x or equivalent merit

8 continued	(b) Using $$\cos A\cos B = \frac{1}{2}[\cos(A-B)+\cos(A+B)]$$ $$\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) = \frac{1}{2}\left[\cos\left(\frac{3x+x}{2}-\frac{3x-x}{2}\right)+\cos\left(\frac{3x+x}{2}+\frac{3x-x}{2}\right)\right]$$ $$= \frac{1}{2}(\cos x+\cos 3x)$$ $$\therefore \cos 3x+\cos x = 2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)$$ $$\cos 3x-\cos 2x+\cos x = 0$$ $$2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right)-\cos 2x = 0$$ $$2\cos 2x\cos x - \cos 2x = 0$$ $$\cos 2x(2\cos x-1) = 0$$ $$\cos 2x = 0 \text{ or } 2\cos x - 1 = 0$$ $$2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$ $$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$ or $2\cos x = 1$ $$\cos x = \frac{1}{2}$$ $$x = \frac{\pi}{3} \text{ or } \frac{5\pi}{3}$$	3	3 marks for all solutions with valid working 2 marks for a correct method used to solve the equation, some missing answers or a small error 1 marks for progress toward using the compound angle results to rearrange equation

8 continued	(c) (i) Let $u = x$ $u' = 1$ $v = \cos^{-1} x$ $v' = -\dfrac{1}{\sqrt{1-x^2}}$ $\dfrac{d}{dx} x \cos^{-1} x = vu' + uv'$ $= \cos^{-1} x - \dfrac{x}{\sqrt{1-x^2}}$	1	1 marks for correct derivative
	(ii) $\int \cos^{-1} x - \dfrac{x}{\sqrt{1-x^2}}\, dx = x \cos^{-1} x$ $\int \cos^{-1} x\, dx = x \cos^{-1} x + \int \left(\dfrac{x}{\sqrt{1-x^2}}\right) dx$ Let $u = 1 - x^2$ $du = -2x\ dx$ $\int \left(\dfrac{x}{\sqrt{1-x^2}}\right) dx = -\dfrac{1}{2}\int -\dfrac{du}{\sqrt{u}}$ $= -\dfrac{1}{2}\int u^{-\frac{1}{2}}\ \mathrm{du}$ $= -\dfrac{1}{2}\left(2u^{\frac{1}{2}}\right)$ $= -\sqrt{u}$ $= -\sqrt{1-x^2}$ $\therefore \int \cos^{-1} x\, dx = x \cos^{-1} x - \sqrt{1-x^2} + C$	2	2 marks for correct answer showing appropriate integration 1 mark for correct use of (i) with some progress towards answer

8 continued	(d) (i) initial velocity $= 202\ \text{km/h}$ $= 56.\dot{1}\ \text{m/s}$ $v\ (0) \approx 56\ \cos 5° i + 56 \sin 5° j$ $\approx 55.8i + 4.9j$ $\underset{\sim}{\mathbf{a}}\ (t) = -9.8\underset{\sim}{\mathbf{j}}$ $\underset{\sim}{\mathbf{v}}\ (t) = \int -9.8\underset{\sim}{\mathbf{j}}\ \text{dt}$ $= -9.8t\underset{\sim}{\mathbf{j}} + C$ $\underset{\sim}{\mathbf{v}}\ (0) = 55.8\underset{\sim}{\mathbf{i}} + 4.9\underset{\sim}{\mathbf{j}}$ $\therefore -9.8(0)\underset{\sim}{\mathbf{j}} + C = 55.8\underset{\sim}{\mathbf{i}} + 4.9\underset{\sim}{\mathbf{j}}$ $C = 55.8\underset{\sim}{\mathbf{i}} + 4.9\underset{\sim}{\mathbf{j}}$ $\underset{\sim}{\mathbf{v}}\ (t) = -9.8t\underset{\sim}{\mathbf{j}} + 55.8\underset{\sim}{\mathbf{i}} + 4.9\underset{\sim}{\mathbf{j}}$ $\underset{\sim}{\mathbf{s}}\ (t) = \int 55.8\underset{\sim}{\mathbf{i}} - 9.8t\underset{\sim}{\mathbf{j}} + 4.9\underset{\sim}{\mathbf{j}}\ \text{dt}$ $= 55.8t\underset{\sim}{\mathbf{i}} - 4.9t^2\underset{\sim}{\mathbf{j}} + 4.9t\underset{\sim}{\mathbf{j}} + C_1$ $= 55.8t\underset{\sim}{\mathbf{i}} + 4.9t(1-t)\underset{\sim}{\mathbf{j}} + C_1$ $\underset{\sim}{\mathbf{s}}\ (0) = 2.8\underset{\sim}{\mathbf{j}}$ $\therefore\quad C_1 = 2.8\underset{\sim}{\mathbf{j}}$ $\underset{\sim}{\mathbf{s}}\ (t) = 55.8t\underset{\sim}{\mathbf{i}} + (4.9t - 4.9t^2 + 2.8)\underset{\sim}{\mathbf{j}}$	3	3 marks for valid proof showing unit conversion and derivation from $a(t)$, including evaluation of constants 2 marks for correct integration of a(t) and v(t) 1 marks for correct unit conversion or one correct integration step
	(ii) When the ball hits the ground its vertical displacement will be zero. $-4.9t^2 + 4.9t + 2.8 = 0$ $t = \dfrac{-4.9 \pm \sqrt{(4.9)^2 - 4\ (-4.9)(2.8)}}{-2(-4.9)}$ $t = -0.4$ or 1.41 Ignore negative solution as $t > 0$ $d = s\ (1.406326967) = 55.8(1.406326967)$ $= 78.47304477\ \text{m}$ $\therefore d > 26.2\ \text{m}$ So, the ball will be called out.	2	2 marks for correct calculation of horizontal displacement when vertical is zero and comparison 1 marks for attempt to find horizontal displacement when vertical is zero.

9 continued	(a) (i) 6**b**-3**a**	1	1 Mark: Correct answer.
	(a) (ii) $\overrightarrow{OX} = 3\boldsymbol{a} + \frac{1}{3}\ (6\boldsymbol{b} - 3\boldsymbol{a})$ $\quad = 2\boldsymbol{a} + 2\boldsymbol{b}$ $\frac{2}{5}\overrightarrow{OY} = \frac{2}{5}(6\boldsymbol{b} + 5\boldsymbol{a} - \boldsymbol{b})$ $\quad = 2\boldsymbol{a} + 2\boldsymbol{b}$ $\therefore \overrightarrow{OX} = \frac{2}{5}\overrightarrow{OY}\ as\ required$	3	3 Marks: Correct answer. 2 Marks: makes some progress 1 Mark: finds one of the vectors required
	(c) (i) Substituting $x = 3$ into $y = f(x)$ gives $y = 1 + 3 + e^3$ So A has coordinates $(3, 4 + e^3)$ A' has coordinates $(4 + e^3, 3)$	1	1 marks for correct answer
	(ii) The tangent at A' will have a gradient that is the reciprocal of the gradient of the tangent at A. $y = 1 + x + e^x$ $y' = 1 + e^x$ At $x = 3$ $y' = 1 + e^3$ $\therefore\ m$ of tangent at $A' = \frac{1}{1 + e^3}$ $y - y_1 = m(x - x_1)$ $y - 3 = \frac{1}{1 + e^3}(x - (4 + e^3))$ $y(1 + e^3) = x - (4 + e^3) + 3(1 + e^3)$ $y = \frac{x - 1 + 2e^3}{1 + e^3}$	2	2 marks for correct gradient leading to correct equation 1 mark for stating gradient relations hip

9 continued	(iii) The tangents will meet on the line $y = x$ due to the inverse functions. $y = \dfrac{x - 1 + 2e^3}{1 + e^3}$ ① $y = x$ ② $x = \dfrac{x - 1 + 2e^3}{1 + e^3}$ $x(1 + e^3) = x - 1 + 2e^3$ $x + xe^3 = x - 1 + 2e^3$ $xe^3 = -1 + 2e^3$ $x = \dfrac{2e^3 - 1}{e^3}$ $y = \dfrac{2e^3 - 1}{e^3}$ $\therefore$ point of intersection $= \left(\dfrac{2e^3 - 1}{e^3}, \dfrac{2e^3 - 1}{e^3}\right)$ **Alternate Method of solving the two Tangents simultaneously is more time consuming, but gives the same result, but likely to be unsimplified.** **Abbreviated working follows below**	2	2 marks for correct answer with working 1 mark for recognising intersection is on $y = x$ and starting to work towards solution or attempting to solve simultaneously with an error or incomplete working
	Equation of tangent at A is $y - (4 + e^3) = (1 + e^3)(x - 3)$ $y = (1 + e^3)x - 2e^3 + 1$ Solve with tangent at A' $(1 + e^3)x - 2e^3 + 1 = \dfrac{x - 1 + 2e^3}{1 + e^3}$ which leads to $x = \dfrac{2e^6 + 3e^3 - 2}{2e^3 + e^6} = \dfrac{(2e^3 - 1)(e^3 + 2)}{e^3(2 + e^3)} = \dfrac{2e^3 - 1}{e^3}$ $y = (1 + e^3)\dfrac{2e^6 + 3e^3 - 2}{2e^3 + e^6} - 2e^3 + 1$ $= \dfrac{2e^6 + 3e^3 - 2 + 2e^9 + 3e^6 - 2e^3}{e^3(2 + e^3)} + \dfrac{(-2e^3 + 1)(2e^3 + e^6)}{e^3(2 + e^3)}$ $= \dfrac{2e^6 + 3e^3 - 2 + 2e^9 + 3e^6 - 2e^3}{e^3(2 + e^3)} + \dfrac{-4e^6 - 2e^9 + 2e^3 + e^6}{e^3(2 + e^3)}$ $y = \dfrac{2e^6 + 3e^3 - 2}{e^3(2 + e^3)} = \dfrac{(2e^3 - 1)(e^3 + 2)}{e^3(2 + e^3)} = \dfrac{2e^3 - 1}{e^3}$ Which gives the same answer as previous method, but accept unsimplified answer of: $\left(\dfrac{2e^6 + 3e^3 - 2}{2e^3 + e^6}, \dfrac{2e^6 + 3e^3 - 2}{2e^3 + e^6}\right)$		

Markers Comment Year 12 Extension mathematics 2021

Question 6

a)i) and ii) were well done

b) Generally well done but some students still forgetting restriction of x from denominator of equation, in this case x can not equal 1. A few had issues with testing or lack of testing which shows a need to review this again.

c) Well done except a few students made calculation errors or didn't notice the negative in front of cos so must be in second quadrant.

d) i) Some students struggle with identifying how to draw an inverse function or didn't read then need to restrict the domain and drew an inverse relation so lost the marks.

ii) Some students mixed up reciprocal of the inverse function with a reciprocal of the reciprocal. More time needs to be spent recognising correct notation. Also some students didn't recognise it was the reciprocal of the y-values so y=3 became y=1/3 and y=0 became y=1/0 which didn't exist, so a asymptote must exist. More revision of this is required.

Question 7

a) Most students can do this induction question. A few has very poor technique (e.g. do not know when to factorise), clearly from a lack of practice.

(bi)This part is very well done. Those who has gone on to solve for values of C has a better idea of the sketch in part (ii)

(bii) Need to show that the gradient is infinite at (-1,0) and (2,0). Otherwise students might lose 1m. Some students could show the exact y-intercept which is impressive.

(ci) A number of students do not realise that they need to do a substitution and lose the mark.

(cii) About 8-10 students have not attempted this part at all. Notable reasons are lack of exposure/practice and time management issues. Some of the response were excellent especially for those who have used definite integration.

A well done question on the whole.

Question 8

a)i) Well done

ii) Quite a few errors in this like forgetting to find the boundary in terms of x, forgetting pi or when expanding y squared, incorrect algebra. Substitution of the expression from part (i) was generally well done.

b) i) 2 methods available, fastest method was using product identities but using addition identities still works. Students who used either of these correctly generally got full marks. Most students couldn't simplify their trig identities correctly to get any marks. If they could some students lost marks as forgot 2x means double the domain as well.

c)i) Generally well done.

ii) This question was poorly done as quite a few students did not show their working. Working is very important in any examination setting. Some students confused the question with inverse trig, need to work on recognising the difference.

d)
i) Students were able to apply the correct process to generate the displacement vector. However, some students did not convert to the correct units.
ii) This was done well.

Question 9

a i) Quite well done
a ii) Genreally well done but with some students not able to find the correct vectors for OX or OY, and trying to fudge answer as if they have proven the required statement

bi) Quite well done
b ii) A lot of students are not able to find the gradient of the inverse by using the reciprocal relationship
b iii) poorly answered, less than 2 students actually can solve the tangent of the inverse with y = x. Vast majority tried to find the intersection between the tangents of f(x) and its inverse and ended in nowhere